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1.Uniformity Test:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| n | ppf(k=10) | Chi-squared  (k=10) | Status  (k=10) | ppf(k=20) | Chi-squared  (k=20) | Status  (k=20) |
| 20 | 14.68 | 7.08 | Not rejected | 27.20 | 17.21 | Not rejected |
| 500 | 14.68 | 2.0 | Not rejected | 27.20 | 7.68 | Not rejected |
| 4000 | 14.68 | 6.95 | Not rejected | 27.20 | 17.34 | not rejected |
| 10000 | 14.68 | 7.08 | Not rejected | 27.20 | 17.21 | Not rejected |

For large n, χ 2 will have an approximate chi-square distribution with k − 1 degrees of freedom under the null hypothesis: H0 = Ui ’s are IID random variables. We reject this hypothesis at level α if χ 2 > χ2 (k−1,1−α) where χ 2 (k−1,1−α) is the upper 1 − α critical point of the chi-square distribution with k − 1 degrees of freedom.

Here for all the test it is “not rejected”.So the uniformity test passes.

2.Serial test:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| n | d=2,k=4  (ppf,chi-squared) | d=2,k=4  (status) | d=2,k=8  (ppf,chi-squared) | d=2,k=8  (status) | d=3,k=4  (ppf,chi-squared) | d=3,k=4  (status) | d=3,k=8  (ppf,chi-squared) | d=3,k=8  (status) |
| 20 | 22.30,  12.8 | Not rejected | 77.74,38.40 | Not rejected | 77.74,27.20 | Not rejected | 552.37,161.60 | Not rejected |
| 500 | 22.30, 130.04 | rejected | 77.74,152.83 | rejected | 77.74,240.96 | rejected | 552.37,378.944 | Not rejected |
| 4000 | 22.30,1013.45 | rejected | 77.74,1039.26 | rejected | 77.74,1803.45 | rejected | 552.37,1948.52 | rejected |
| 10000 | 22.30,2512.94 | rejected | 77.74,2536.47 | rejected | 77.74,4467.70 | rejected | 552.37,4620.29 | rejected |

The serial test, is really just a generalization of the chi-square test to higher dimensions. If the Ui ’s were really IID U(0, 1) random variates, the nonoverlapping d-tuples should be IID random vectors distributed uniformly on the d-dimensional unit hypercube, [0, 1]d

If the individual Ui ’s are correlated, the distribution of the d-vectors Ui will deviate from d-dimensional uniformity; thus, the serial test provides an indirect check on the assumption that the individual Ui ’s are independent. For example, if adjacent Ui ’s tend to be positively correlated, the pairs (Ui , Ui+1) will tend to cluster around the southwest-northeast diagonal in the unit square. Finally, it should be apparent that the serial test for d >3 could require a lot of memory to tally the kd values of fj1j2…jd.

Here ,when n is smaller the serial test is “not rejected”.If we increase n the serial test rejects.

**3.Run test:**

|  |  |  |  |
| --- | --- | --- | --- |
| n | ppf | R | status |
| 20 | 10.64 | 0.9399 | Not rejected |
| 500 | 10.64 | 2.39 | Not rejected |
| 4000 | 10.64 | 7.27 | Not rejected |
| 10000 | 10.64 | 5.77 | Not rejected |

The third empirical test we consider, the runs (or runs-up) test, is a more direct test of the independence assumption. (In fact, it is a test of independence only; i.e., we are not testing for uniformity in particular.) We examine the Ui sequence (or, equivalently, the Zi sequence) for unbroken subsequences of maximal length within which the Ui ’s increase monotonically; such a subsequence is called a run up.

Here all the results are “not rejected”.So the run up is distributed independently.

**4.Corelation test:**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| n | J=1  ppf,|Aj| | J=1  status | J=3  ppf,|Aj| | J=3  status | J=5  ppf,|Aj| | J=5  status |
| 20 | 1.64,1.49 | Not rejected | 1.64,0.18 | Not rejected | 1.64,1.33 | Not rejected |
| 500 | 1.64,0.42 | Not rejected | 1.64,1.46 | Not rejected | 1.64,1.32 | Not rejected |
| 4000 | 1.64,1.10 | Not rejected | 1.64,1.98 | rejected | 1.64,0.53 | Not rejected |
| 10000 | 1.64,0.98 | Not rejected | 1.64,1.32 | Not rejected | 1.64,1.30 | Not rejected |

Under the null hypothesis that ρj = 0 and assuming that n is large, it can be shown that the test statistic

has an approximate standard normal distribution. This provides a test of zero lag j correlation at level α, by rejecting this hypothesis s if |Aj | > Z1-α/2.

The test should probably be carried out for several values of j, since it could be, for instance, that there is no appreciable correlation at lags 1 or 5, but there is dependence between the Ui ’s at lag 3, due to some anomaly of the generator.